Controlled regularity at future null infinity from past asymptotic initial data: The wave equation

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Outline

Motivation

Conformal scattering, a notion fully developed by Friedlander in the 1980s, described how two independent works in the 1960s; the scattering theory of Lax-Phillips and the ideas of Penrose's conformal compactification of asymptotically simple spacetimes, could be combined.

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With the notion of the conformal boundary now available, one could ask the question, *how is the past and future asymptotic data of massless fields related to each other?*

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Take an asymptotically flat spacetime. Then the conformal boundary $\mathscr{I}$ is a null hypersurface. So, a conformal scattering problem here takes the form of a characteristic initial value problem with data prescribed on $\mathscr{I}$.

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Another question, *how does the gravitational radiation of physical objects and the regularity of the past conformal boundary* $\mathscr{I}^-$ *affect the structure of solutions at the future* $\mathscr{I}^+$*?*
One answer: There is *polyhomogeneous* behaviour towards the conformal boundary and spatial infinity. Einstein's field equations are very non-linear and are partially responsible for this.

Geometric set-up

Here, we encounter *the problem of spatial infinity*. If we wish to study the wave equation as a characteristic initial value problem with data on past null infinity $\mathscr{I}^-$, then we need a better representation of spatial infinity than the one acquired by Penrose compactification.

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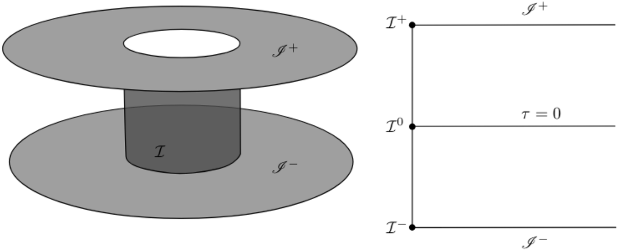

Figure: Friedrich's cylinder at spatial infinity.

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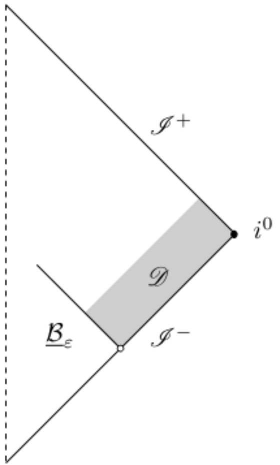

Figure: The domain of interest for the wave equation.

Main results

Theorem

Given data on past null infinity $\mathscr{I}^-$ and on a short incoming null hypersurface that is sufficiently regular, solutions to the wave equation are sufficiently smooth at future null infinity. In particular, the multipolar structure of the characteristic data determines the behaviour towards the past critical set $\mathscr{I}^+$.

More explicitly, the expansion admitted looks like

$$\phi = \sum_{p'=0}^{N} \frac{1}{p'!} \phi^{(p')} (\tau, t)\, \rho^{p'} + C^{m+1,\alpha}.$$

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Moreover, the expansion one can obtain in the F-gauge can be shown to be analytic if one utilises the characteristic nature of the cylinder and analyses the solutions to the transport equations.

Future directions

A similar result of this kind would be excellent to understand on the Schwarzschild or Kerr backgrounds to see what effects the curvature has on the estimates used to prove such a result. And how much the conclusion of the theorem changes, i.e. can sufficiently smooth data give rise to polyhomogeneous behaviour?

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On a different note, one may ask if the set-up was restricted from the beginning. To this end, we may approach this problem in a new manner by combining the b-calculus that is used by Hintz, Vasy and others together with Friedrich's cylinder to gain deeper insights into polyhomgeneous behaviour in this corner of spacetime.

A picture speaks more...