Non-linearities after the collision of two black holes

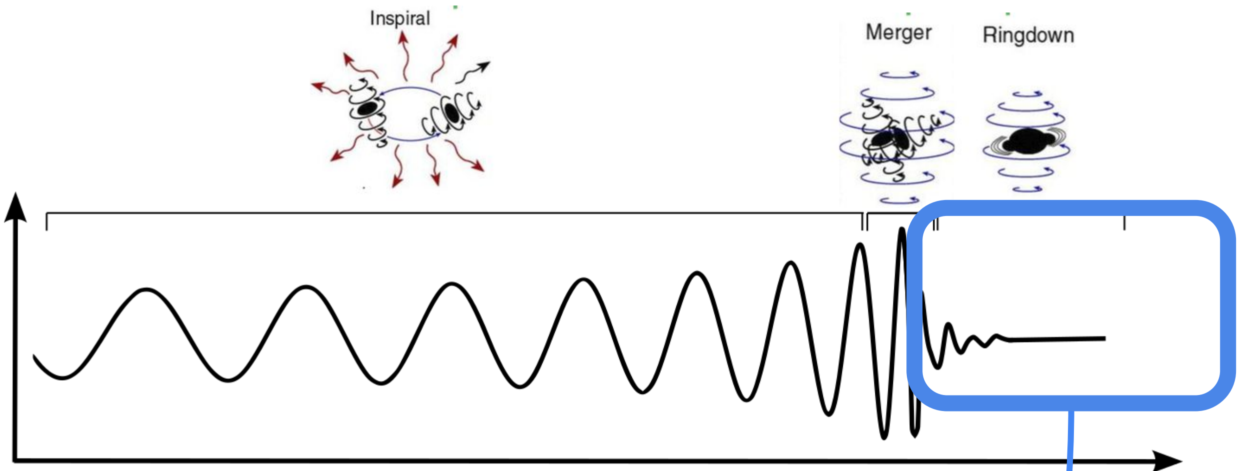
Béatrice Bonga - 24 January 2025

[Khera, Ribes Metidieri, BB, Jiménez Forteza, Krishnan, Poisson, Pook-Kolb, Schnetter, Yang, PRL, arXiv:2306/11142 + Bourg, Panoso Macedo, Spiers, Leather, BB, Pound, arXiv:2405.10270, PRL]

Radboud University

Gravitational waves make possible

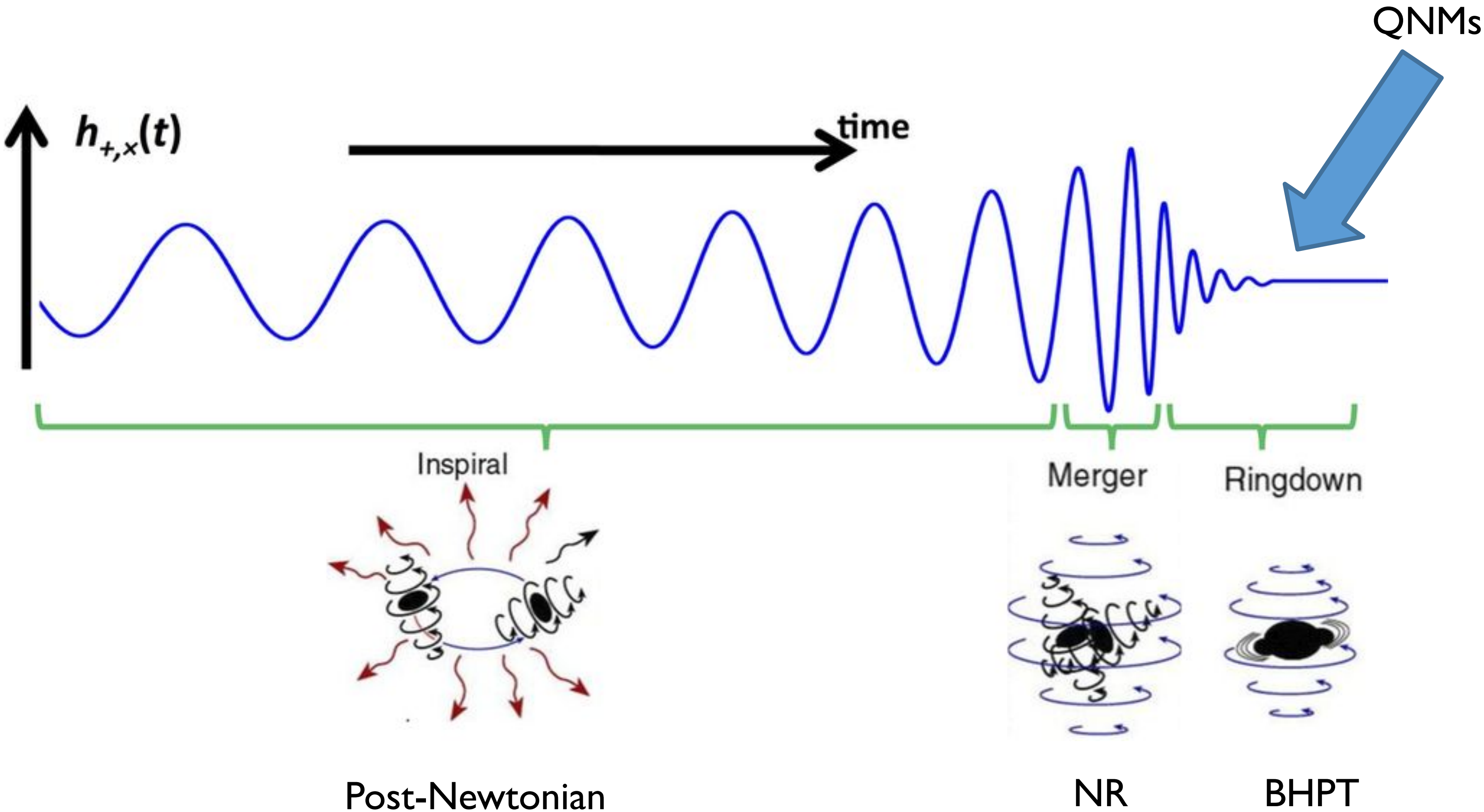
1) Test of gravity in the **strong** & **dynamical** regime



2) Test of the **non-linear** nature of gravity

observation = linear + non-linear *+ noise*

Gravitational waves from black hole mergers



Quasi-normal modes

Mathematical description

$$h_+ + i\,h_\times = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} h_{lm}(t,r)\;\; {}_2Y_{lm}(\theta,\phi)$$

spin-weighted
spherical harmonic

$$h_{lm}(t,r) = \frac{1}{r}\sum_{n=0}^{N} A_{lmn} e^{-i\omega_{lmn}(t-t_0)+\Phi_{lmn}}$$

Depend on the details
of the "hammer"

Frequencies and damping times

$$\omega_{lmn} = \omega_{lmn}^{R} + i\,\omega_{lmn}^{I} = 2\pi f_{lmn} + \frac{i}{\tau_{lmn}}$$

Depends on three integers:

$$l = 2,3, \ldots$$
$$-l < m < l$$
$$n = 0,1,2, \ldots$$

Damping time

→ ***Black hole spectroscopy!***

Going beyond linear order…

	Linear order
Frequency	$\omega_{nlm}^{(1)}$
Phase	$\phi_{nlm}^{(1)}$
Amplitude	$\mathcal{A}_{nlm}^{(1)}$

data analysis:
free parameters

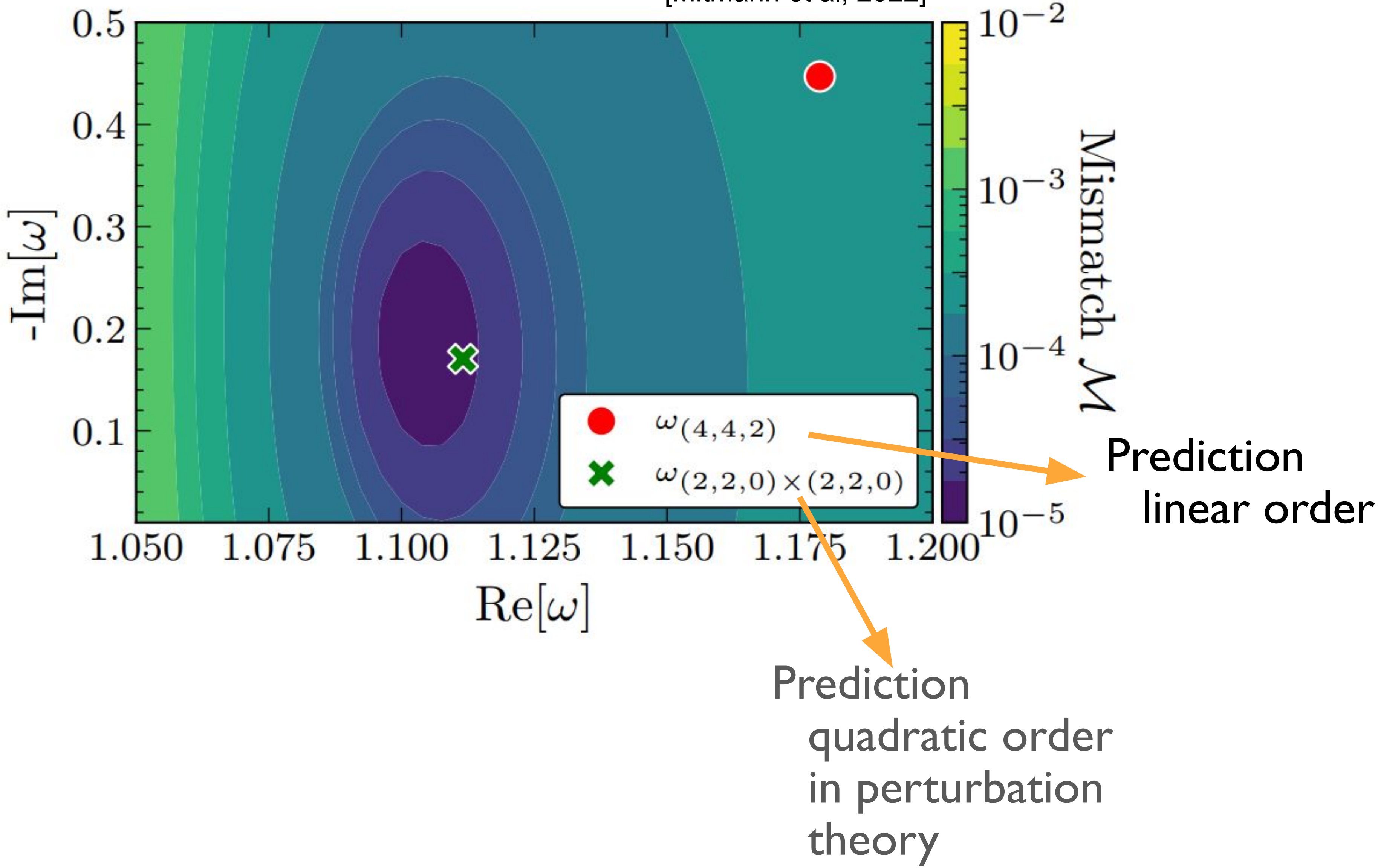
Going beyond linear order…

	Linear order	Second order
Frequency	$\omega^{(1)}_{nlm}$	$\omega^{(2)}_{NLM} = \omega^{(1)}_{nlm} + \omega^{(1)}_{n'l'm'}$ — "quadratic" frequencies are fingerprinted
Phase	$\phi^{(1)}_{nlm}$	$\phi^{(2)}_{NLM} = \phi^{(1)}_{nlm} \pm \phi^{(1)}_{n'l'm'} + d^{NLM}_{nlm\times n'l'm'}(M,a)$
Amplitude	$\mathcal{A}^{(1)}_{nlm}$	$\mathcal{A}^{(2),NLM}_{nlm\times n'l'm'} = c^{NLM}_{nlm\times n'l'm'}(M,a)\mathcal{A}^{(1)}_{nlm}\mathcal{A}^{(1)}_{n'l'm'}$

data analysis: free parameters

Non-linear model preferred @ infinity

[Mitmann et al, 2022]



So why do I think this is exciting?

Implications for observations:

$$h^{obs} = h^{linear} + h^{non-linear}$$

but frequencies are “finger-printed” with an order in perturbation theory!

Can we also model the black hole horizon with QNMs?

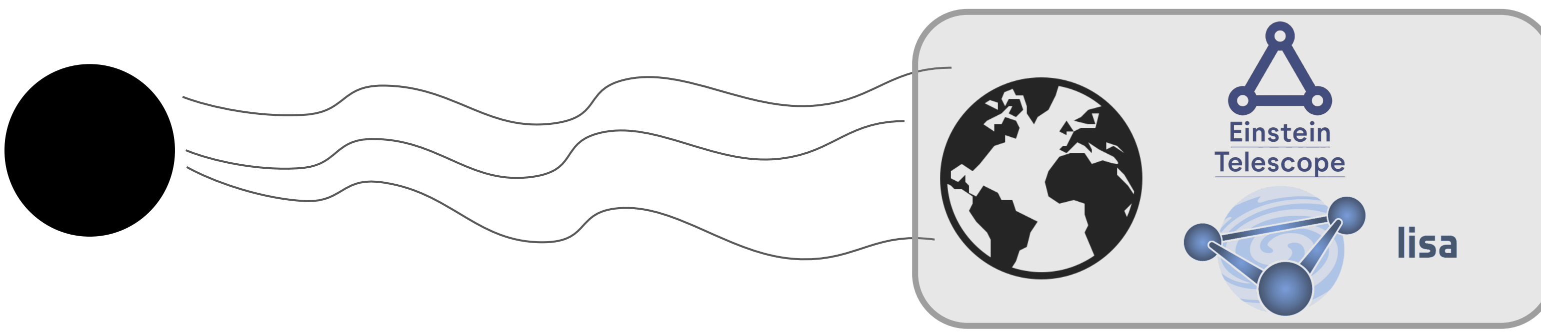
Horizon should be more non-linear, but not too crazy

→ easier to find quadratic QNMs

Horizon is strong field regime

→hopeless to try to find any QNMs

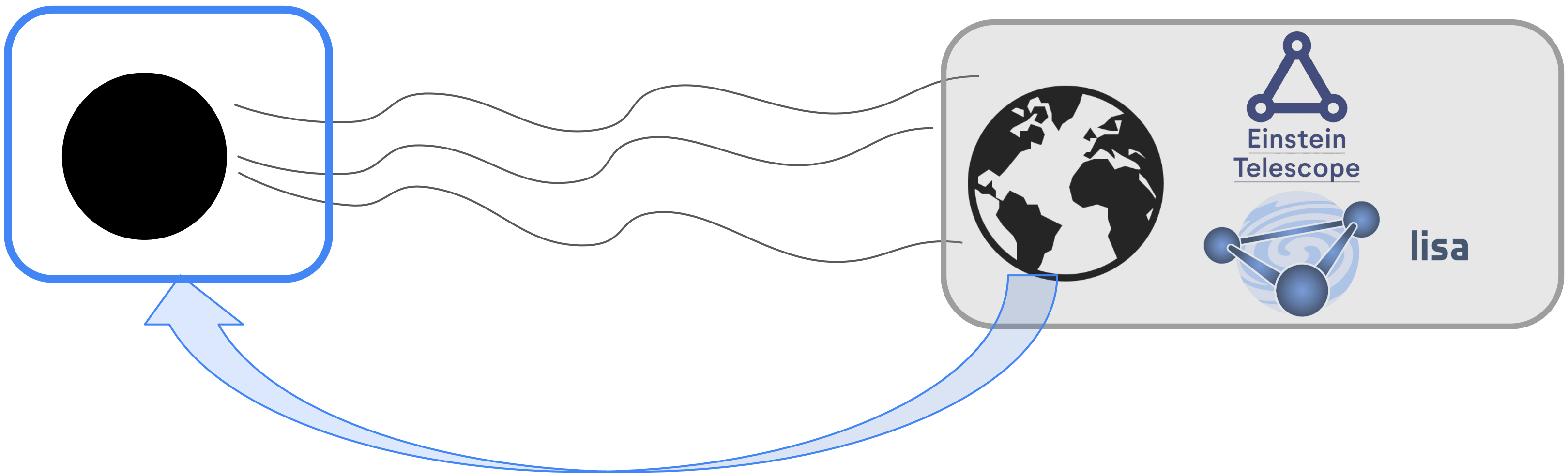
Why care about the horizon?



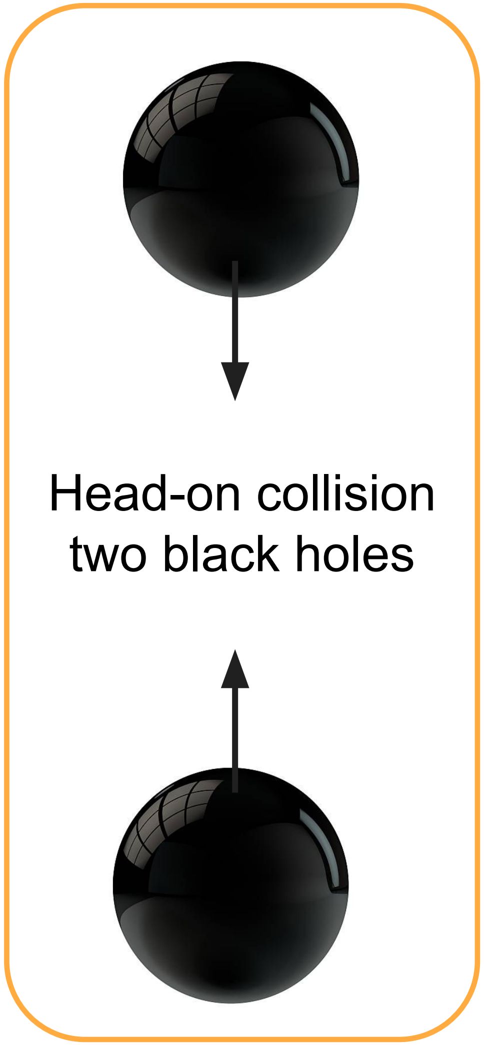
Gravitational waves…

are interesting because of their origin!

Black hole tomography [see presentation Ariadna Ribes Metidieri]



Two sets of simulations using the Einstein Toolkit

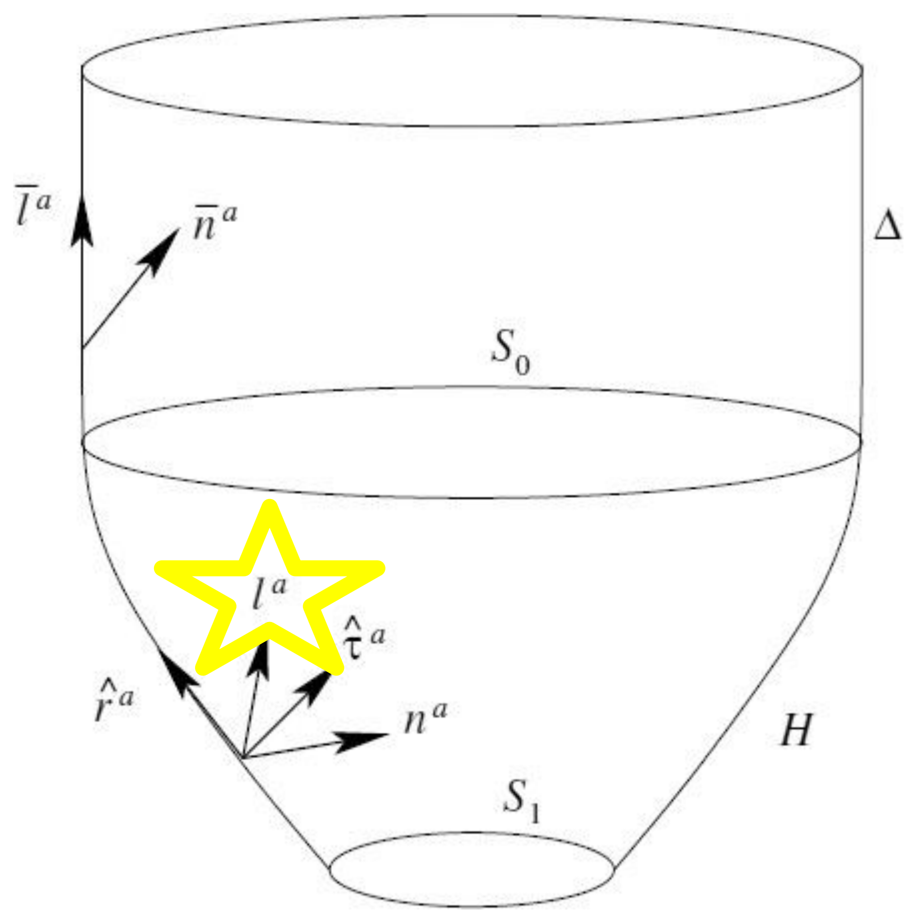


(1) Resulting BH is non-rotating
(2) Axisymmetric simulations ⟶ no m=0 modes
(3) High resolution near horizon (but poor near infinity)

- Unboosted: m1,m2,P=0
- Boosted: m1=m2, P≠ 0

⇨ linear amplitudes 10x bigger

Shear at the horizon



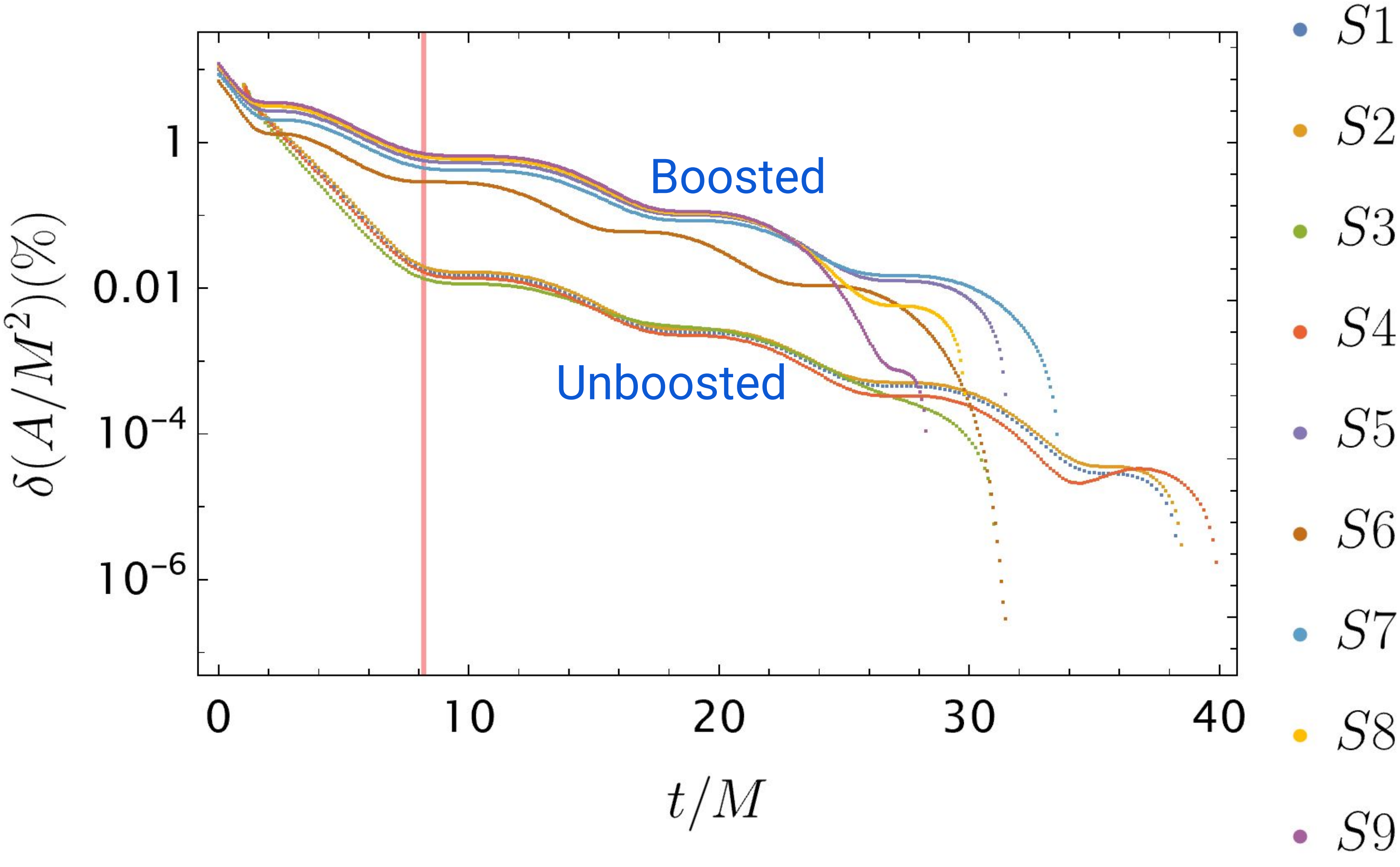
Choice of time

Time ⟷ Definition of frequency

Disclaimer: We simply use the simulation time.

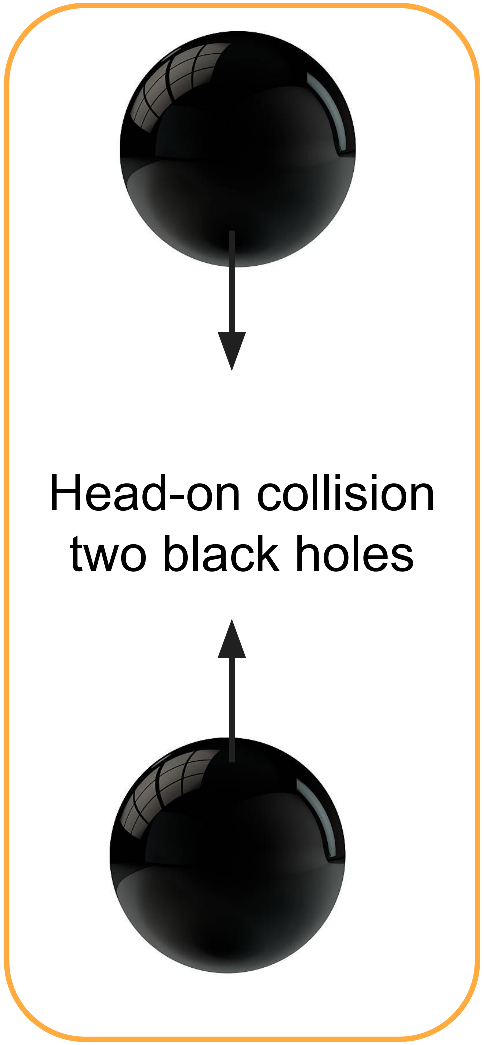
Same issue at infinity!

Ringdown: Mass changes ≤ 1 %



We take $t_{ringdown} = 8.2$ M

Two sets of simulations using the Einstein Toolkit

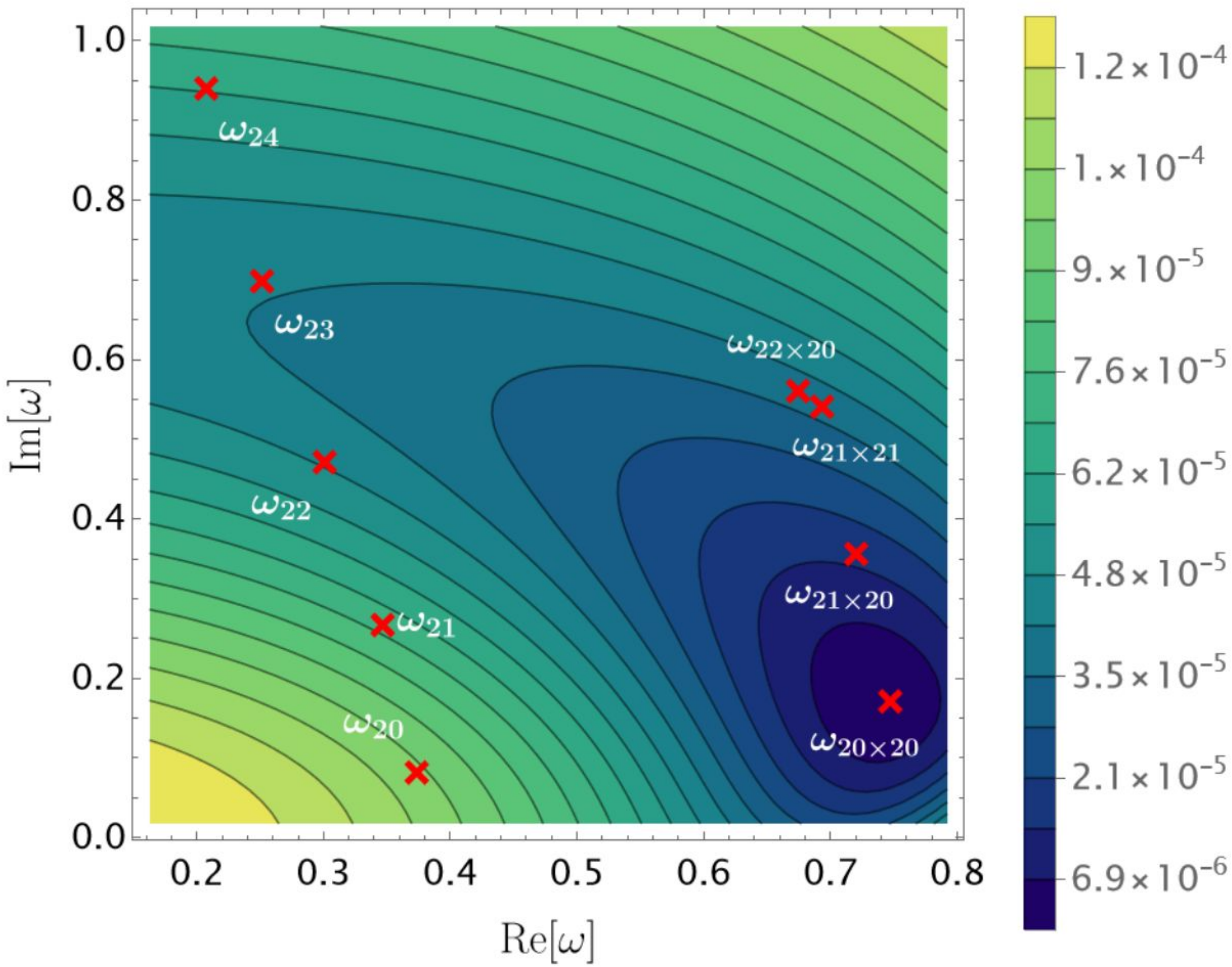


- Unboosted: m1,m2,P=0
  - ⇨ model with 4 tones
- Boosted: m1=m2, P≠ 0
  - ⇨ linear amplitudes 10x bigger
  - ⇨ model with 3 tones

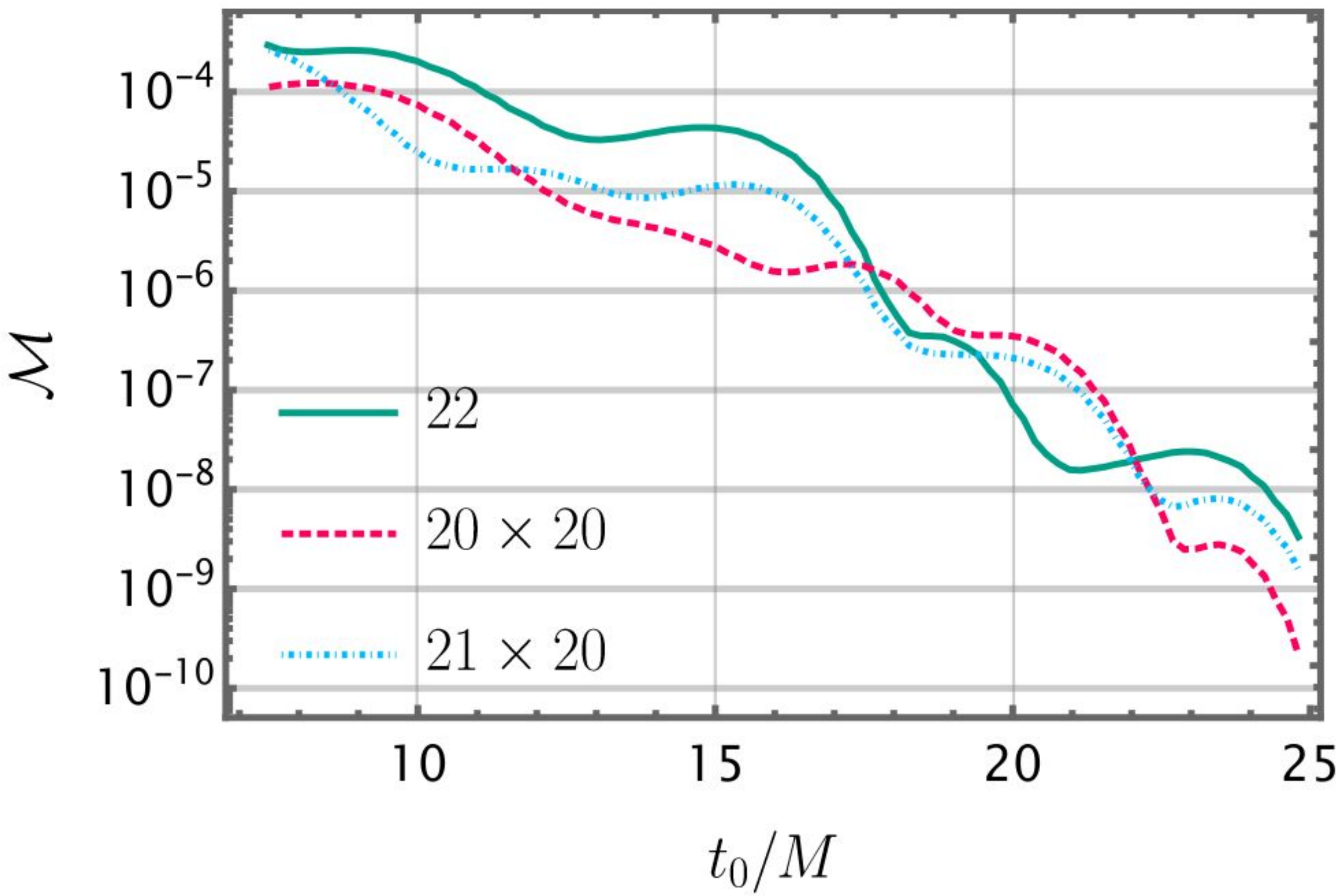
l=2,4,6,... are only non-zero

Notation: $\omega_{lmn} \longrightarrow \omega_{ln}$

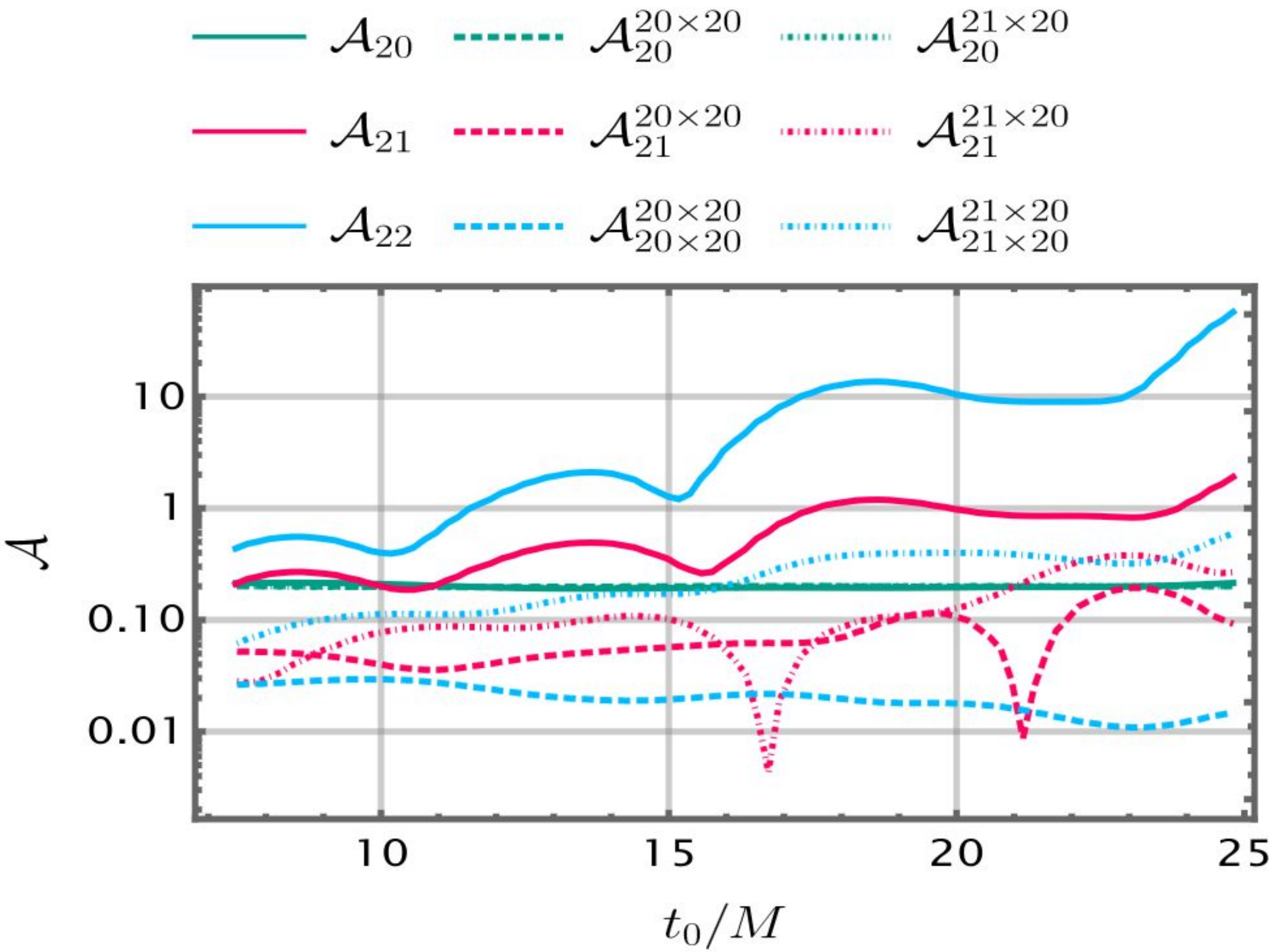
Mismatch after fixing ω_{200} and ω_{201}



Mismatch after fixing ω_{200} and ω_{201}

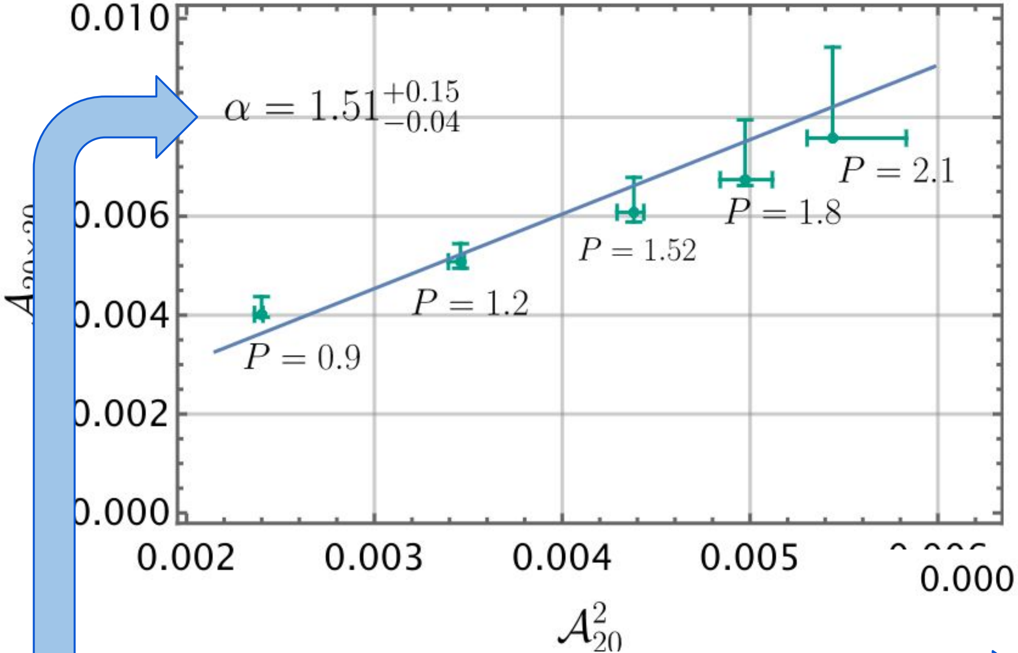


Stability amplitude

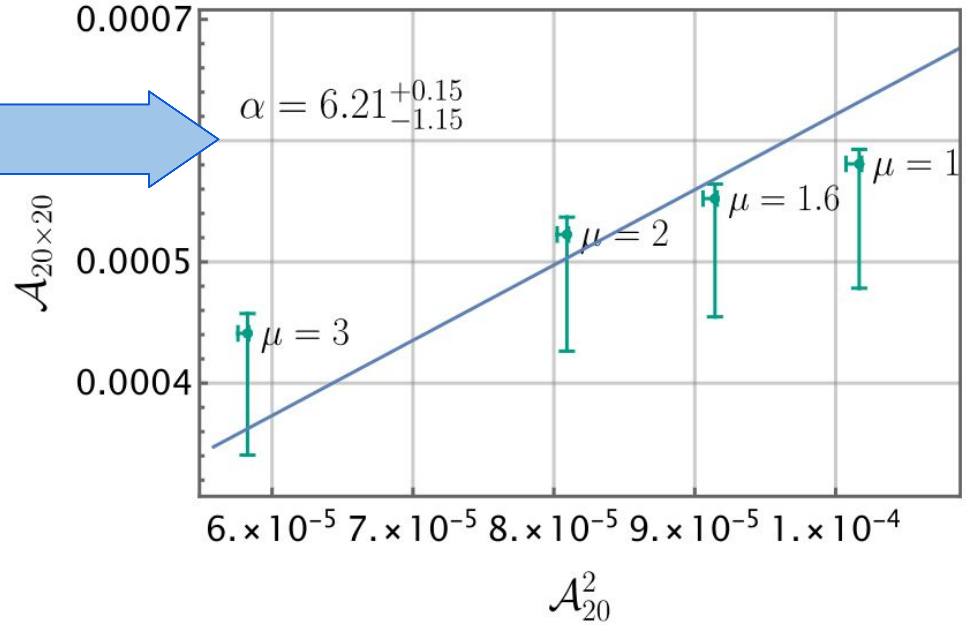


Amplitude relation

Boosted



Unboosted

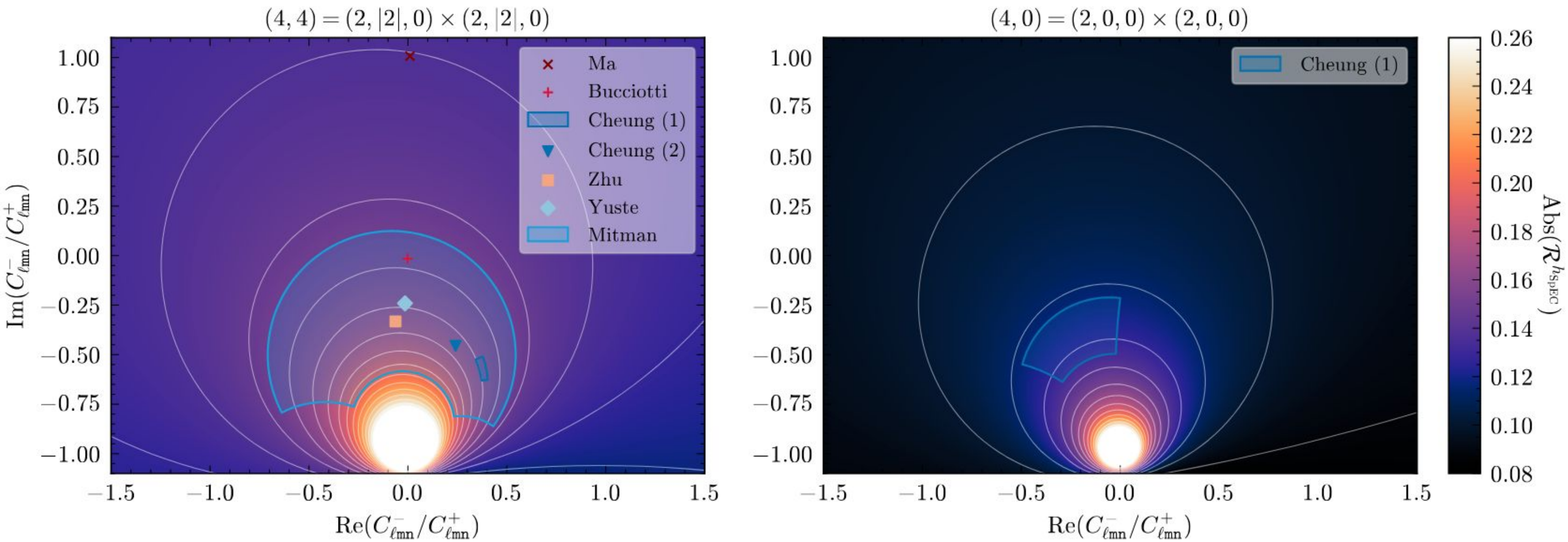


Puzzle: Why are these slopes different?

Amplitude relation is not initial data-independent!

$$C^{-}_{lmn} / C^{+}_{lmn}$$

$$\mathcal{A}^{(2),NLM}_{nlm\times n'l'm'} = c^{NLM}_{nlm\times n'l'm'}(M,a)\,\mathcal{A}^{(1)}_{nlm}\mathcal{A}^{(1)}_{n'l'm'}$$

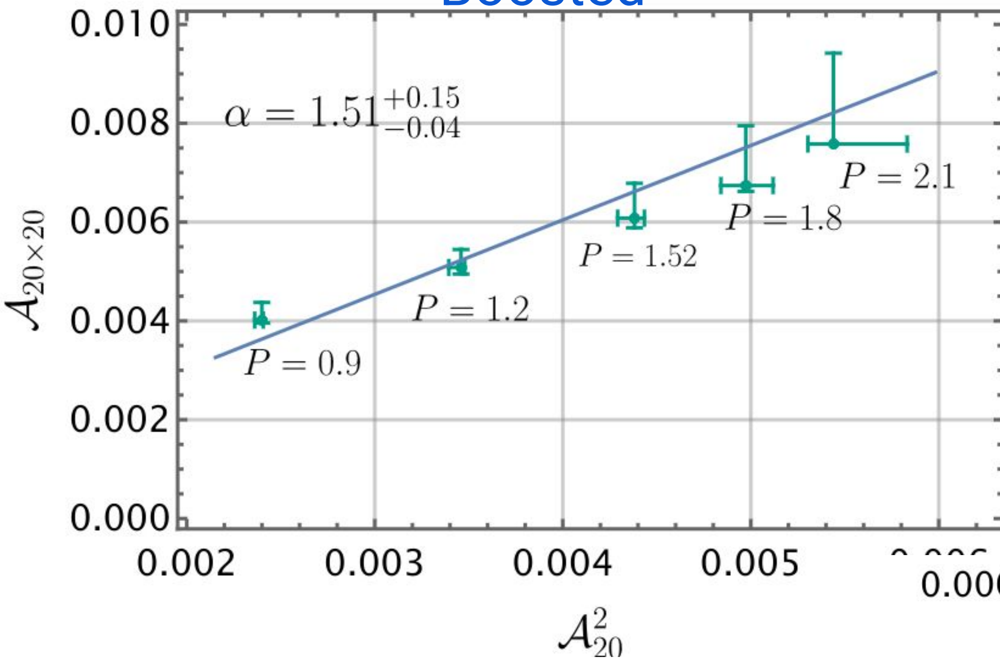


The ratio of odd/even parity linear parent modes is also important

[see presentation Patrick Bourg]

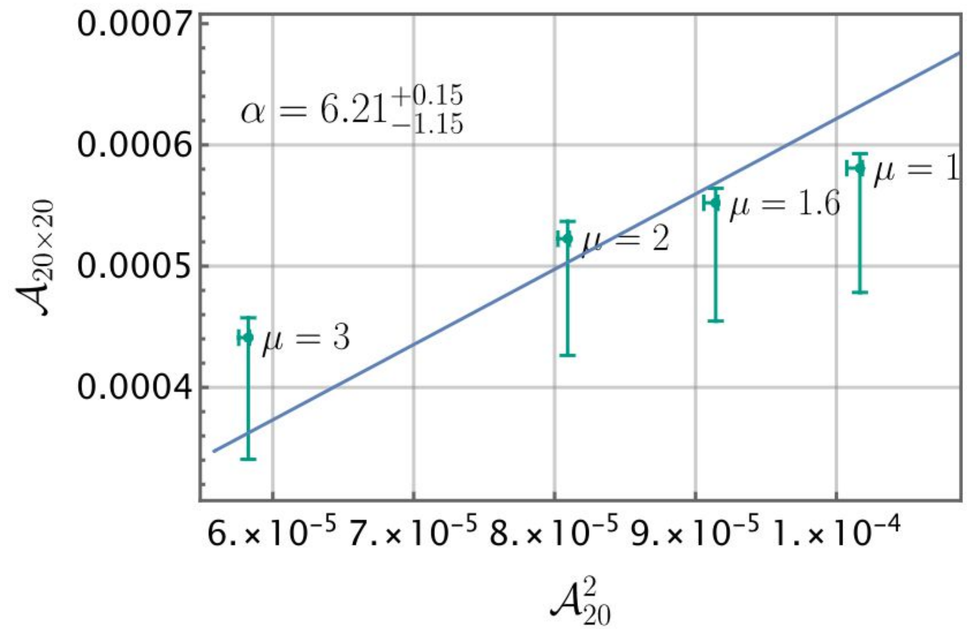
Amplitude relation explained?

Boosted



Up-down symmetry
no even modes!

Unboosted



No symmetry
both odd and even
modes excited

Other l-modes

Mode	$\omega_{ln\times l'n'}$	Boosted (α)	Unboosted (α)
$l=2$	$\omega_{20\times 20}$	$1.51^{+0.15}_{-0.04}$	$6.21^{+0.15}_{-1.15}$
$l=4$	$\omega_{20\times 20}$	$0.73^{+0.06}_{-0.33}$	-
	$\omega_{20\times 40}$	$2.6^{+0.26}_{-0.26}$	-
$l=6$ *	$\omega_{20\times 40}$	$1.78^{0.53}_{-0.74}$	-
	$\omega_{20\times 60}$	$2.52^{+1.29}_{-0.59}$	-
	$\omega_{20\times 40}$	$1.78^{0.44}_{-0.65}$	-
	$\omega_{40\times 40}$	$2.82^{+1.5}_{-0.62}$	-

Conclusion

- ★ Quadratic QNMs fit the shear (and multipole) data at the horizon better than models with overtones
  - lower mismatch
  - more stable amplitudes wrt changes in starting time
  - closer to the optimal frequency
  - amplitude relation is satisfied

- ★ Some of the same (quadratic) modes found at horizon and infinity

- ★ Observations of quadratic modes very likely with ET & LISA, so the future is bright!

Thanks for listening