# 14th Central European Relativity Seminar

February 14-16, 2024, Tübingen

## Schedule

### Wednesday, February 14

10:30 - 11:30 Registration

11:30 - 13:20 Lunch break

13:20 - 13:30 Opening

13:30 - 14:30 **Melanie Graf** (University of Hamburg) – *Singularity theorems from a mathematical perspective*

14:30 - 15:00 Coffee break

15:00 - 15:20 **Barbora Bezdekova** (Charles University, Czech Republic) – *Deflection of light around a spherically symmetric object in a moving plasma medium*

15:20 - 15:40 **Liam Urban** (University of Vienna) – *Stable Big Bang formation for the Einstein scalar-field Vlasov system*

15:40 - 16:00 **John Östör** (Radboud University, Wigner RCP) – *Construction of inhomogeneous cosmological initial data on compact surfaces*

16:00 - 16:20 Coffee break

16:20 - 16:40 **Mario Hudelist** (University of Vienna) – *Curvature-Induced Corrections to Rytov's Law in Optical Fibers*

16:40 - 17:00 **Christian Krueger** (University of Tuebingen) – *Rapidly rotating neutron stars: Universal relations and EOS inference*

### Thursday, February 15

9:30 - 10:30 **Piotr T. Chruściel** (University of Vienna) – *Mass in 2+1 dimensions*

10:30 - 11:00 Coffee break

11:00 - 11:20 **Albachiara Cogo** (Universität Tübingen) – *Equipotential Photon Surface Uniqueness in Electrovacuum*

11:20 - 11:40 **Mario Misas Arcos** (Universität Tübingen) – *Seeking codimension-2 STCMC-surfaces in Minkowski spacetime*

11:40 - 12:00 **Leonardo Garcia Heveling** (Universität Hamburg) – *Global hyperbolicity through the eyes of the null distance*

12:00 - 14:00 Lunch break

14:00 - 14:20 **Finnian Gray** (University of Vienna) – *Homogeneous Symmetry Operators in Kerr–NUT–AdS Spacetimes*

14:20 - 14:40 **Aravindhan Srinivasan** (Charles University & IM, CAS Prague) – *Charging Kerr-Schild spacetimes in higher dimensions*

14:40 - 15:00 **Ádám Marozsi** (Eötvös Loránd Uni. & Wigner RCP) – *Study of the constraint equations in conformally rescaled non-physical spacetimes*

15:00 - 15:30 Coffee break

15:30 - 15:50 **Francisco Fernández-Álvarez** (University of the Basque Country) – *On the degrees of freedom of gravitational radiation with a positive cosmological constant*

15:50 - 16:10 **Saradha Senthil Velu** (University of Tübingen) – *A note on the Arnowitt-Deser-Misner Angular Momentum*

16:10 - 16:30 **Philip Schwartz** (University of Hannover) – *Teleparallel Newton–Cartan gravity*

## Friday, February 16

9:30 - 10:30 **Peter Hintz** (ETH Zurich) – *Gluing black holes along timelike geodesics*

10:30 - 11:00 Coffee break

11:00 - 11:20 **Wan Cong** (University of Vienna) – *On characteristic gluing*

11:20 - 11:40 **Markus Wolff** (KTH Stockholm) – *Null Mean Curvature flow along the de Sitter lightcone*

11:40 - 12:00 **Sharmila Gunasekaran** (Radboud University) – *Rigidity results on near horizon geometries*

# Abstracts

## Talks

### Deflection of light around a spherically symmetric object in a moving plasma medium

*Barbora Bezdekova, Charles University, Czech Republic, baja@etranslator.biz*

During recent years the deflection of light rays propagating around compact gravitational objects (e.g., black holes, neutron stars) has frequently been studied when the object is surrounded by a medium of dispersive and refractive properties, i.e., plasma. The behaviour analysis of such system is usually based on using the Hamiltonian approach, where the equations of motion can easily be found. However, in the most of the analytical approaches the medium in the vicinity of the compact gravitational source is assumed to be static; typically, the cold plasma approximation is considered. In our study we go beyond this limit and assume a radially falling medium which does not have to be cold plasma. We derive the light deflection angle around a general spherically symmetric object. Moreover, we show the general form of the Hamiltonian as a function of radial momentum component which has to be used in order to find an analytical solution. We also study a system when the medium with a nonzero phi-component of the velocity is assumed.

### Mass in 2+1 dimensions

*Piotr T. Chruściel, University of Vienna, piotr.chrusciel@univie.ac.at*

I will review the definition of mass for 2+1 dimensional spacetimes, and of 2-dimensional general relativistic initial data sets, with emphasis on its unusual behaviour under asymptotic symmetries.

### Equipotential Photon Surface Uniqueness in Electrovacuum

*Albachiara Cogo, Universität Tübingen, albachiara.cogo@uni-tuebingen.de*

Equipotential Photon Surfaces are timelike and totally geodesic hypersurfaces in a static electrovacuum spacetime such that the lapse function and the electric potential are spatially constant at each time. They are very interesting objects from the physical point of view and they have mathematical properties that allow us to analyse them adapting techniques developed for the study of static Black Holes. As a result of a joint project with Borghini and Cederbaum (arXiv:2401.05253), I will discuss static electrovacuum spacetimes in presence of a connected Equipotential Photon Surface, providing a characterization of the Reissner–Nordstrm spacetime. For this purpose, an approach via potential theory developed by Agostiniani and Mazzieri will be adopted, allowing not to assume a regular foliation of the spacetime by the lapse function, necessary for some other methods. In addition, this strategy yields a characterization of the spatial slices of the equipotential photon surfaces not only in sub-extremal Reissner–Nordstrm spacetimes, but also in the extremal and super-extremal cases (up to a restiction on the range of radii), which have not been fully tackled in the existing literature.

**On characteristic gluing**

*Wan Cong, University of Vienna, wcong@uwaterloo.ca*

In a series of work, Aretakis, Czimek, and Rodnianski have studied the gluing of two sets of initial data to the Einstein equations along a null surface. By solving the linearized gluing problem around Minkowski and using the implicit function theorem, they have shown that two sets of $C^2$ sphere data sufficiently close to Minkowski can be glued together along a null surface up to a ten dimensional obstruction space. In this talk, I will present a generalisation of this result to (i) higher dimensions, (ii) include the cosmological constant and (iii) the gluing of higher regularity $C^k$ data.

**On the degrees of freedom of gravitational radiation with a positive cosmological constant**

*Francisco Fernández-Álvarez, University of the Basque Country, francisco.fernandez@ehu.eus*

Results towards the isolation of the radiative degrees of freedom of the gravitational field with a positive cosmological constant in full General Relativity are put forward. On three-dimensional Riemannian manifolds, a class of differential operators associated with triads of orthonormal forms is proposed. The space of all such operators is shown to have the structure of an affine space where each point is labelled by 2 functions. Based on results by Friedrich and using a recent characterisation of gravitational radiation in the presence of a positive cosmological constant, the 2 coordinates of this space are understood as half of the radiative degrees of freedom at infinity. Remarkably, they determine utterly the presence of gravitational radiation in space-times with algebraically-special rescaled Weyl tensor at the conformal boundary.

### Global hyperbolicity through the eyes of the null distance

*Leonardo Garcia Heveling, Universitt Hamburg,*
*leonardo.garcia@uni-hamburg.de*

Can one encode the causal information of a spacetime in a metric space structure? In 2016, Sormani and Vega conjectured how this may be done. In this talk, I will present some recent results confirming that the answer is yes. Not only can the causal relation be encoded, but also the condition of global hyperbolicity can be characterized via completeness of the corresponding metric space. This is joint work with Annegret Burtscher.

### Singularity theorems from a mathematical perspective

*Melanie Graf, University of Hamburg, melanie.graf@uni-hamburg.de*

The classical singularity theorems of R. Penrose and S. Hawking from the 1960s are beautiful examples of mathematical results in Lorentzian Geometry with wide physical relevance for General Relativity showing that any spacetime with a smooth Lorentzian metric satisfying certain energy conditions and causality assumptions must be geodesically incomplete. Despite their great success these classical theorems still had and have some drawbacks both in their assumptions and conclusions. Focusing on the assumptions side of the picture I will review the classical theorems and some recent mathematical progress allowing us to relax some of the usual demands placed on the metric regularity as well as the classical pointwise energy conditions, which makes the theorems applicable to a wider class of physically relevant situations.

### Homogeneous Symmetry Operators in Kerr–NUT–AdS Spacetimes

*Finnian Gray, University of Vienna, finnian.gray@univie.ac.at*

It is well known that the Kerr–NUT–AdS spacetimes possess hidden symmetries encoded in the so-called principal Killing–Yano tensor. These hidden symmetries underlie the remarkable integrability and separability properties of test particles and fields in these spacetimes. In this talk, focusing on the four-dimensional case, I will present a number of symmetry operators for scalar, vector, and tensor perturbations, that are of degree two and homogeneous in the principal tensor. In the scalar case the usual symmetry operators and the resulting separability are recovered. However, it remains to be seen whether such operators can be used to separate the corresponding spin 1 and spin 2 test field equations in these spacetimes.

### Rigidity results on near horizon geometries

*Sharmila Gunasekaran, Radboud University, sharmila.gunasekaran@ru.nl*

I will present results on the rigidity of near-horizon geometries (NHGs) with a cosmological constant, extending the results to the larger domain of quasi-Einstein metrics. It was established by Dunajski-Lucietti that the Kerr-AdS NHG is the unique NHG on a 2-sphere. Time permitting, I will present some partial deformability results for this metric. This is joint work with Eric Bahuaud, Hari Kunduri and Eric Woolgar based on Lett Math Physics Vol 112 - No. 116, Vol 114 - No. 8 and arXiv:2310.06186.

### Gluing black holes along timelike geodesics

*Peter Hintz, ETH Zurich, peter.hintz@math.ethz.ch*

Given a smooth globally hyperbolic spacetime satisfying the Einstein vacuum equations and a timelike geodesic, I will describe the construction of a family of metrics which are close to the original metric away from the geodesic, but which near it describe a Kerr black hole of very small mass traveling along the geodesic. These metrics are formal solutions of the Einstein field equations in that they satisfy the field equations to all orders in the mass of the small black hole. In the special case that we apply this construction to a subextremal Kerr or Kerr-de Sitter black hole spacetime and we choose for the geodesic one that crosses the future event horizon, we can thus describe extreme mass ratio mergers to all orders in perturbation theory.

### Curvature-Induced Corrections to Rytov's Law in Optical Fibers

*Mario Hudelist, University of Vienna, a01414109@unet.univie.ac.at*

According to Rytov's law, the polarization vector of light follows a Fermi-Walker transport equation in optical fibers. Recent advancements in theory propose a modification to Rytov's law due to fiber bending. The aim of this talk is to further extend these predictions from flat to curved space-time. This involves perturbatively solving Maxwell's equations under the assumption that the wavelength is significantly shorter than the fiber radius, as well as the characteristic length-scales of the ambient space-time. This results in a coupling of the polarization vector to the Riemann curvature tensor.

**Rapidly rotating neutron stars: Universal relations and EOS inference**

*Christian Krueger, University of Tuebingen, christian.krueger@tat.uni-tuebingen.de*

Astrophysical neutron stars may be oblate due to fast rotation and the deviation from sphericity must be taken into account when doing equation of state (encoding nuclear physics) inference from neutron star observations. Solving the coupled elliptic partial differential equations describing the equilibria of neutron stars in full general relativity is a computationally rather expensive task. In order to circumvent this issue, we provide so-called universal relations that allow to estimate to percent accuracy the moment of inertia $I$ (and other bulk quantities) of fast rotating neutron stars from the knowledge of mass $M_\star$, radius $R_\star$ and moment of inertia $I_\star$ of an associated non-rotating neutron star; these quantities are computationally easy to access by solving the well-known TOV equations. The proposed universal relations then facilitate inference codes based on Markov chain Monte Carlo with which future, high-precision measurements coming from electromagnetic and gravitational wave observations can be analysed. We show results of such an MCMC inference code based on mock data as a proof of concept. This is joint work with Prof. Kostas Kokkotas.

**Study of the constraint equations in conformally rescaled non-physical spacetimes**

*Ádám Marozsi, Eötvös Loránd Uni. & Wigner RCP, marozsia06@gmail.com*

It is well known that by applying an $n+1$ decomposition in general relativity, Einstein's equations can be split into constraint and evolution equations. In particular, it is also known that the constraint equations propagate if the evolution equations hold. The constraint equations can also be solved by applying either the conformal [L,Y] or the evolutionary [R] formulation of the constraints. It is of fundamental interest to know whether analogues of these results can hold in spacetimes obtained by the conformal compactification procedure of Penrose. The answer turns out to be affirmative. In particular, we have shown that once an $n+1$ decomposition of non-physical spacetimes has been performed, with splitting the non-physical Einstein equations into constraint and evolution equations, the "constraints" do propagate, notably this happens without restricting the trace of the physical sources. It is also shown that the constraints in the non-physical setup can be solved as evolutionary systems. Finally, issues concerning the interpretation of the obtained results are discussed.

[L]: Lichnerowicz A: Lintegration des Equations de la Gravitation Relativiste et le Probleme des n Corps, J. Math. Pures Appl., 23, 39-63 (1944), DOI: `http://www.numdam.org/item/JMPA_1944_9_23__37_0/`

[Y]: York J W: Role of conformal three-geometry in the dynamics of gravitation, Phys. Rev. Lett. 28, 1082-1085 (1972), DOI: `https://doi.org/10.1103/PhysRevLett.28.1082`

[R]: Racz, I. Constraints as evolutionary systems. Class. Quant. Grav. 33, 015014. (2016), DOI: `https://doi.org/10.1088/0264-9381/33/1/015014`

**Seeking codimension-2 STCMC-surfaces in Minkowski spacetime**

*Mario Misas Arcos, Universität Tübingen,*
*mario.misas-arcos@student.uni-tuebingen.de*

The aim of the talk is to present the main results attained within the author's Master's Thesis "On the existence of constant spacetime mean curvature 2-surfaces in Minkowski spacetime", supervised by Professor Carla Cederbaum. In the first place, the innovative technique to find codimension-2 STCMC-surfaces in Minkowski proposed by Professor Cederbaum shall be illustrated. This will be followed by a portrayal of all the STCMC-surfaces found using said method, delving into their respective geometric and topological properties. Finally, any connection between these results and existent work on the topic, such as that by Professor Shing-Tung Yau, will be highlighted, as well as the areas of research that may follow the path explored by this project.

**Construction of inhomogeneous cosmological initial data on compact surfaces**

*John Östör, Radboud University, Wigner RCP, janos.ostor@gmail.com*

We present a new method to construct near-FLRW lambda-vacuum initial data sets on Cauchy surfaces with topology $S^3$ using the parabolic-hyperbolic form of the constraint equations. We apply a fully-spectral treatment in the angular sector, while a pseudo-spectral solver is used in the remaining transverse direction. The gravitational wave content of the solutions is controlled by one of the freely specifiable variables.

**Teleparallel Newton–Cartan gravity**

*Philip Schwartz, University of Hannover, philip.schwartz@itp.uni-hannover.de*

We discuss a teleparallel version of Newton–Cartan gravity. This theory arises as a formal large-speed-of-light limit of the teleparallel equivalent of general relativity (TEGR). Thus, it provides a geometric formulation of the Newtonian

limit of TEGR, similar to standard Newton–Cartan gravity being the Newtonian limit of GR. We show how by a certain gauge-fixing the standard formulation of Newtonian gravity can be recovered.

### A note on the Arnowitt-Deser-Misner Angular Momentum

*Saradha Senthil Velu, University of Tübingen,*
*saradha.senthil-velu@student.uni-tuebingen.de*

In this short talk, we explore the properties of the ADM angular momentum in asymptotically flat initial data sets. We find that the ADM angular momentum does not exhibit conservation in time. This observation challenges the understanding of angular momentum conservation for isolated gravitating systems. We suggest a remedy for this issue. This is early work joint with Carla Cederbaum.

### Charging Kerr-Schild spacetimes in higher dimensions

*Aravindhan Srinivasan, Charles University & IM, CAS Prague,*
*saravindhan7@gmail.com*

Although there are explicit examples of numerical and perturbative solutions of charged rotating black hole solutions in higher dimensional Einstein-Maxwell theory, an explicit exact solution in closed form is unknown. In their famous paper "Black holes in higher dimensional spacetimes," Myers and Perry attempted to obtain a charged version of their black hole solution with one rotation parameter, analogous to Kerr-Newman. The attempt did not result in a charged-black hole but in a weak "no-go" result. Motivated partly by this and recent developments in Kerr-Schild double copy, we study the Kerr-Schild class of spacetimes in higher dimensional Einstein-Maxwell theory, including an arbitrary cosmological constant. Assuming an expanding Kerr-Schild (K-S) vector $\mathbf{k}$ and a vector potential aligned along it, we restrict our study to the charged solutions that can be generated by a K-S transformation of a vacuum K-S solution. In the case of twisting $\mathbf{k}$, we show that a charged solution can exist only when $\mathbf{k}$ is shear-free, thereby extending Myers-Perry's previous "no-go" result. We show that the general twisting-shearfree solutions are charged (A)dS-Taub-NUT solutions with a base space of constant holomorphic sectional curvature. In the non-twisting case, we obtain an example of a shearing Vaidya-like solution with a null electromagnetic field in five dimensions.

**Stable Big Bang formation for the Einstein scalar-field Vlasov system**

*Liam Urban, University of Vienna, liam.urban@univie.ac.at*

While Vlasov matter, as a descriptor of a collisionless self-gravitating gas, has proven itself as a robust model in describing the late universe, the little we know about its past evolution indicates that it becomes unstable toward the past. On the other hand, there have been many recent results showing that scalar field matter can dampen such behaviours in other settings and lead to quiescent Big Bang formation. I will discuss recent joint work with David Fajman, in which we show past nonlinear stability of FLRW solutions to the Einstein scalar-field Vlasov system with non-trivial scalar field matter. Such solutions exhibit stable Kretschmann scalar blow-up, thus becoming geodesically incomplete. Furthermore, the asymptotic behaviour of the Vlasov distribution can be described up to an error controlled by the initial data. This is, to our knowledge, the first past stability without symmetry result that includes Vlasov matter. To ensure that the scalar field sufficiently mitigates potential instabilities generated by Vlasov matter, we crucially exploit a scaling hierarchy between horizontal and vertical derivatives in the Vlasov equation.

**Null Mean Curvature flow along the de Sitter lightcone**

*Markus Wolff, KTH Stockholm, markuswo@kth.se*

We consider null mean curvature flow along the standard lightcone in the de Sitter spacetime. This flow was first studied by Roesch–Scheuer along null hypersurfaces for the detection of MOTS, and independently by the author in the specific case of the standard Minkowski lightcone. Similar to Minkowski case, null mean curvature flow along the de Sitter lightcone can be related to $2d$-Ricci flow in the conformally round case by an appropriate rescaling. Building on this rescaling procedure, we analyse singularity formation, and asymptotic behavior of the flow.

## Posters

**Eicheon properties and dark matter halo around Milky Way Galaxy**

*Sergey Cherkas, Institute for Nuclear Problems BSU, cherkas@inp.bsu.by*

Eicheon is a horizon-free object which appears instead of a black hole in conformally-unimodular metric [1]. It is shown that the eicheon surface allows setting a boundary condition for the vacuum polarization and obtaining a solution describing the dark matter tail in the Milky Way Galaxy [2]. That is, the dark matter in the Milky Way Galaxy is explained as the F-type of vacuum polarization, which could be treated as dark radiation. Surface density of a baryonic galaxy disk is taken into account approximately by smearing the disk over a sphere. This allows the reproduction of the large distance shape of the Milky Way Galaxy rotational curve.

[1] Cherkas, S.L.; Kalashnikov, V.L. Eicheons instead of Black holes. Phys. Scr. 2020, 95, 085009.

[2] Cherkas, S.L.; Kalashnikov, V.L. Rotational Curves of the Milky Way Galaxy and Andromeda Galaxy in Light of Vacuum Polarization around Eicheon Universe 2023, 9(9), 424.