On massive photons in dielectrics

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Joint work with Thomas Mieling, Piotr T. Chruściel

Proca in vacuum

$$\mathcal{L} = -\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta} + \frac{1}{2}\mu^2 A^\alpha A_\alpha$$

$$\partial^\alpha F_{\alpha\beta} + \mu^2 A_\beta = 0 \quad \stackrel{\partial^\beta}{\Longrightarrow} \quad \partial^\beta A_\beta = 0$$

$\eta \ldots$ Minkowski metric
$\mu \ldots$ photon mass
$A \ldots$ vector potential

Proca in dielectric

Gordon metric: $\gamma^{\alpha\beta} = \eta^{\alpha\beta} + (1 - n^2)u^\alpha u^\beta$

$$\mathcal{L} = -\frac{1}{4}\gamma^{\alpha\rho}\gamma^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + \frac{1}{2}\mu^2 \qquad A_\alpha A_\beta$$

$$\gamma^{\alpha\beta}\partial_\alpha F_{\beta\sigma} + \mu^2 \qquad A_\beta = 0 \quad \Longrightarrow \qquad \partial_\alpha A_\beta = 0$$

$\mu\ldots$ photon mass
$A\ldots$ vector potential
$u\ldots$ 4-velocity of medium
$n\ldots$ refractive index

Proca in dielectric

Gordon metric: $\gamma^{\alpha\beta} = \eta^{\alpha\beta} + (1 - n^2)u^\alpha u^\beta$

$$\mathcal{L} = -\frac{1}{4}\gamma^{\alpha\rho}\gamma^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + \frac{1}{2}\mu^2\eta^{\alpha\beta}A_\alpha A_\beta$$

$$\gamma^{\alpha\beta}\partial_\alpha F_{\beta\sigma} + \mu^2\gamma_{\sigma\alpha}\eta^{\alpha\beta}A_\beta = 0 \quad \Longrightarrow \quad \eta^{\alpha\beta}\partial_\alpha A_\beta = 0$$

$\mu\ldots$ photon mass
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Proca in dielectric

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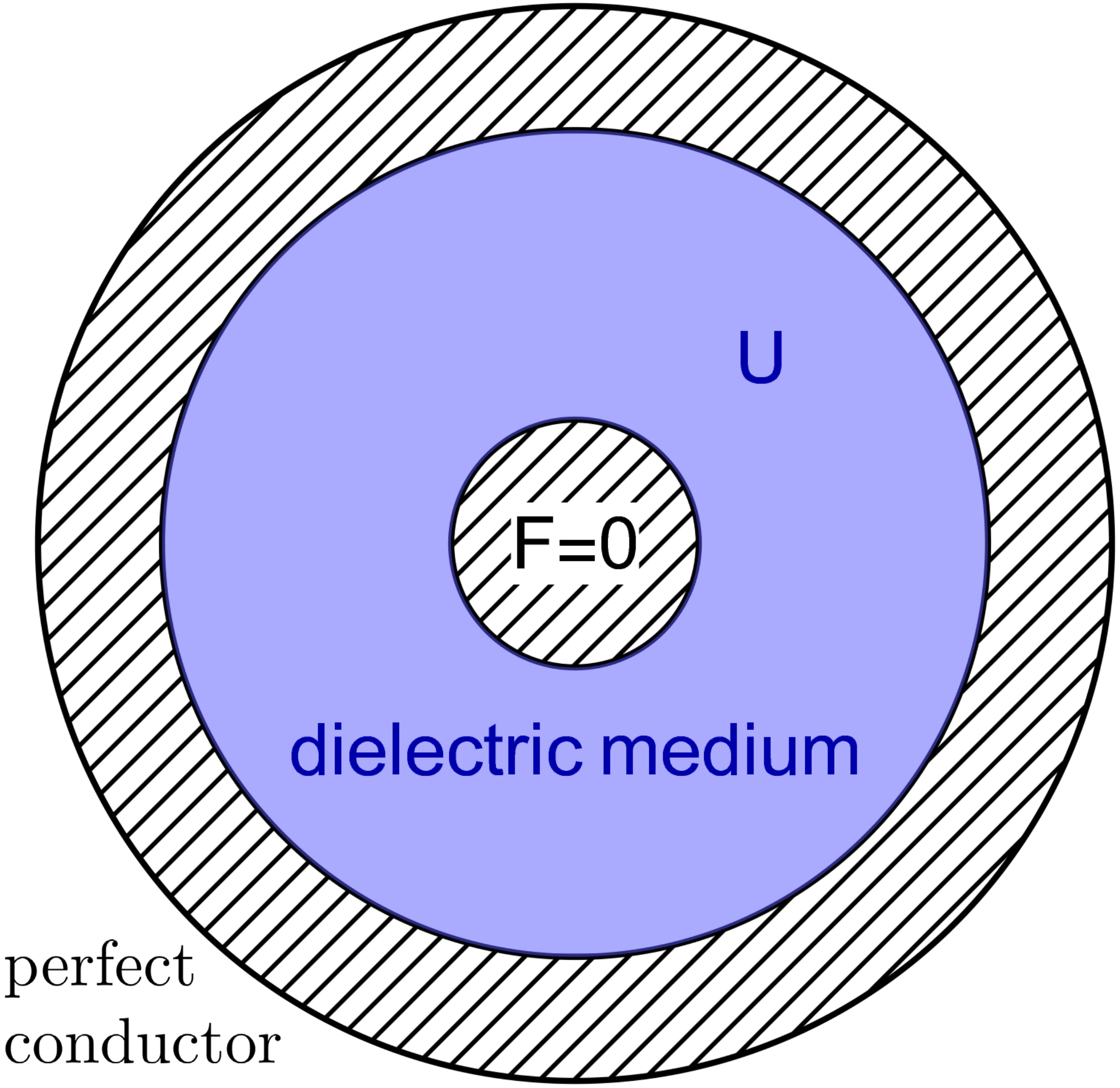
$$\gamma^{\alpha\beta}\partial_\alpha F_{\beta\sigma} + \mu^2 A_\sigma = 0 \quad \Longrightarrow \quad \gamma^{\alpha\beta}\partial_\alpha A_\beta = 0$$

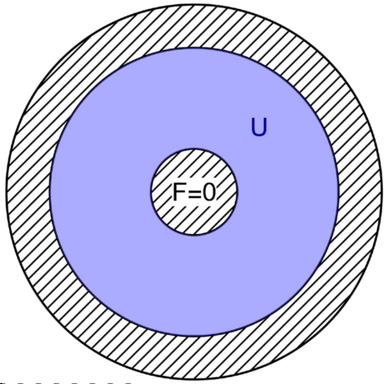
$\mu \ldots$ photon mass
$A \ldots$ vector potential
$u \ldots$ 4-velocity of medium
$n \ldots$ refractive index

Coaxial Waveguide

$$\begin{cases} (\Box_\gamma - \mu^2) A_\alpha = 0 & \text{in } U \\ & \text{on } \partial U \end{cases}$$



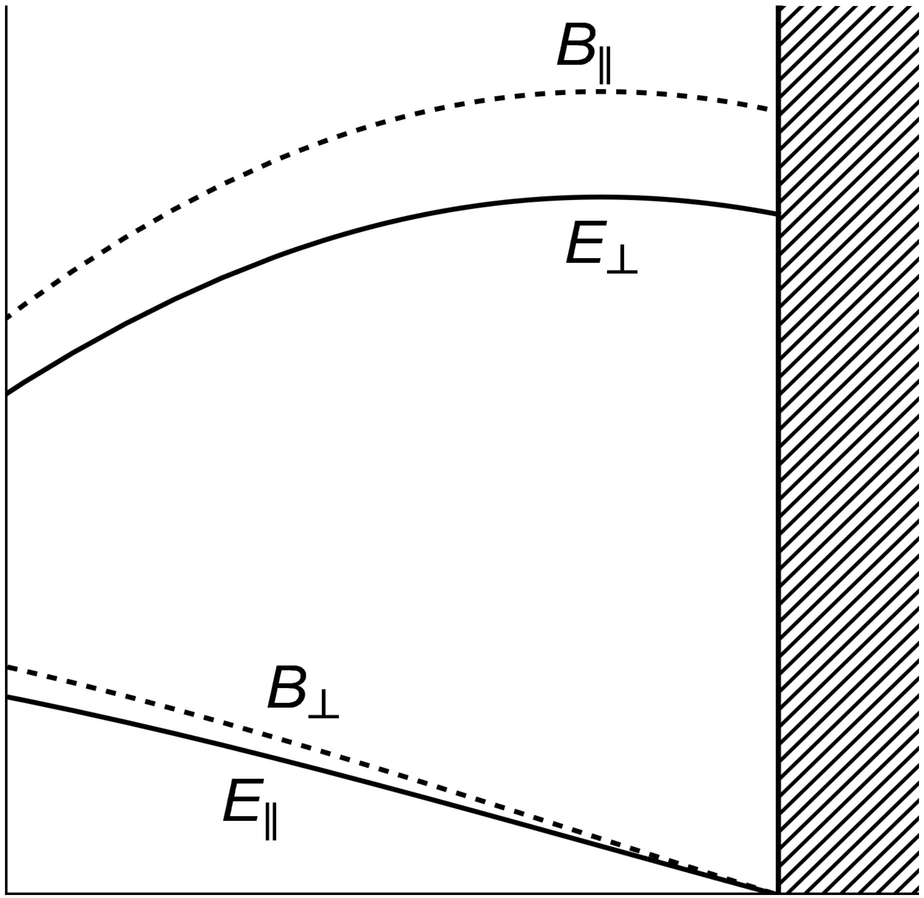
Coaxial Waveguide - Boundary



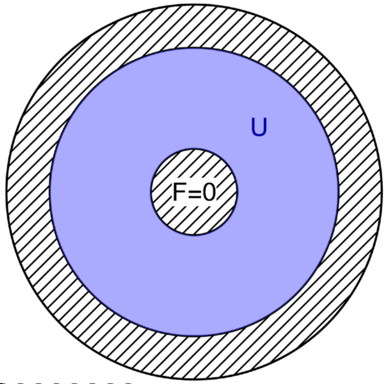
$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ & \text{on } \partial U \end{cases}$$

Maxwell:

$$\nabla \cdot \mathbf{B} = 0 \qquad \qquad \mathbf{n} \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = 0 \quad \Longrightarrow \quad \mathbf{n} \times \mathbf{B} = 0$$



Coaxial Waveguide - Boundary

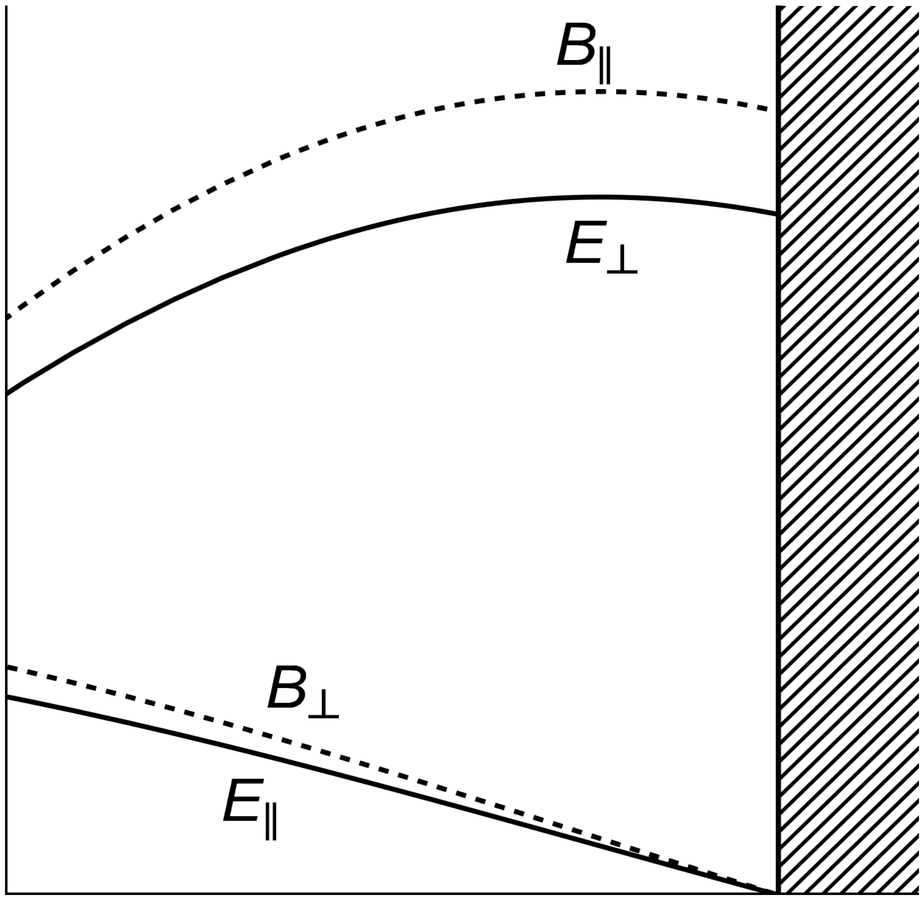


$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ E_\parallel = B_\perp = 0 & \text{on } \partial U \end{cases}$$

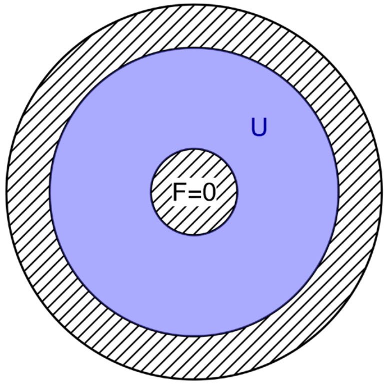
Maxwell:

$$\begin{aligned} \nabla \cdot \mathbf{B} = 0 \\ \nabla \times \mathbf{E} = 0 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \mathbf{n} \cdot \mathbf{B} = 0 \\ \mathbf{n} \times \mathbf{B} = 0 \end{aligned}$$

$$F = 0 \quad \not\Longrightarrow \quad A = 0$$



Coaxial Waveguide - Boundary



$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ E_{||} = B_\perp = 0 & \text{on } \partial U \end{cases}$$

Maxwell:

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$$F = 0 \quad \not\Longrightarrow \quad A = 0$$

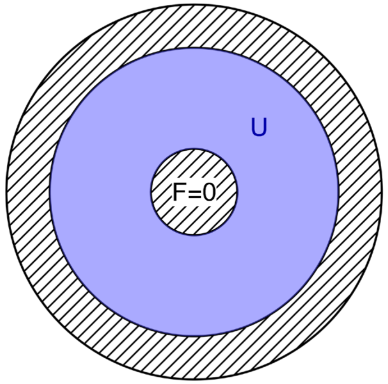
$$\begin{cases} (\Box_\gamma - \mu^2) A_\alpha = 0 & \text{in } U \\ & \text{on } \partial U \end{cases}$$

Proca:

$$\gamma^{\alpha\beta} \partial_\alpha F_{\beta\sigma} + \mu^2 A_\sigma = 0$$

$$F \equiv 0 \quad \Longleftrightarrow \quad A \equiv 0$$

Coaxial Waveguide - Boundary



$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ E_{||} = B_\perp = 0 & \text{on } \partial U \end{cases}$$

Maxwell:

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$$F = 0 \quad \not\Longrightarrow \quad A = 0$$

$$\begin{cases} (\Box_\gamma - \mu^2) A_\alpha = 0 & \text{in } U \\ A_\alpha = 0 & \text{on } \partial U \end{cases}$$

Proca:

$$\gamma^{\alpha\beta}\partial_\alpha F_{\beta\sigma} + \mu^2 A_\sigma = 0$$

$$F \equiv 0 \quad \Longleftrightarrow \quad A \equiv 0$$

$$A_\alpha \text{ continuous}$$

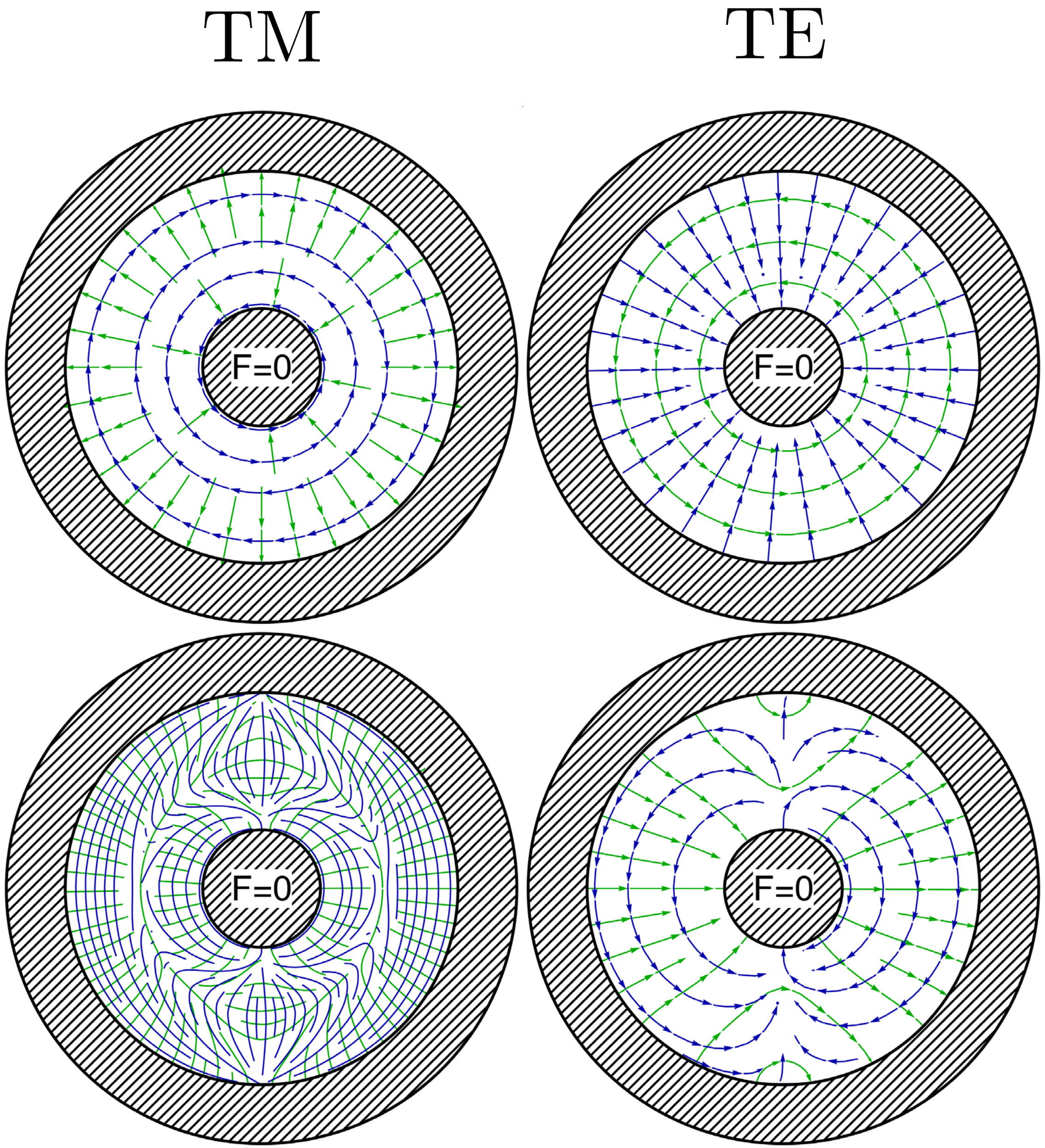
Coaxial Waveguide

$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ E_\parallel = B_\perp = 0 & \text{on } \partial U \end{cases}$$

$\Longrightarrow$

Mode solutions:

TM:	$B_z = 0$ in U
TE:	$E_z = 0$ in U
TEM:	$B_z = E_z = 0$ in U



Coaxial Waveguide

$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{in } U \\ E_\parallel = B_\perp = 0 & \text{on } \partial U \end{cases}$$

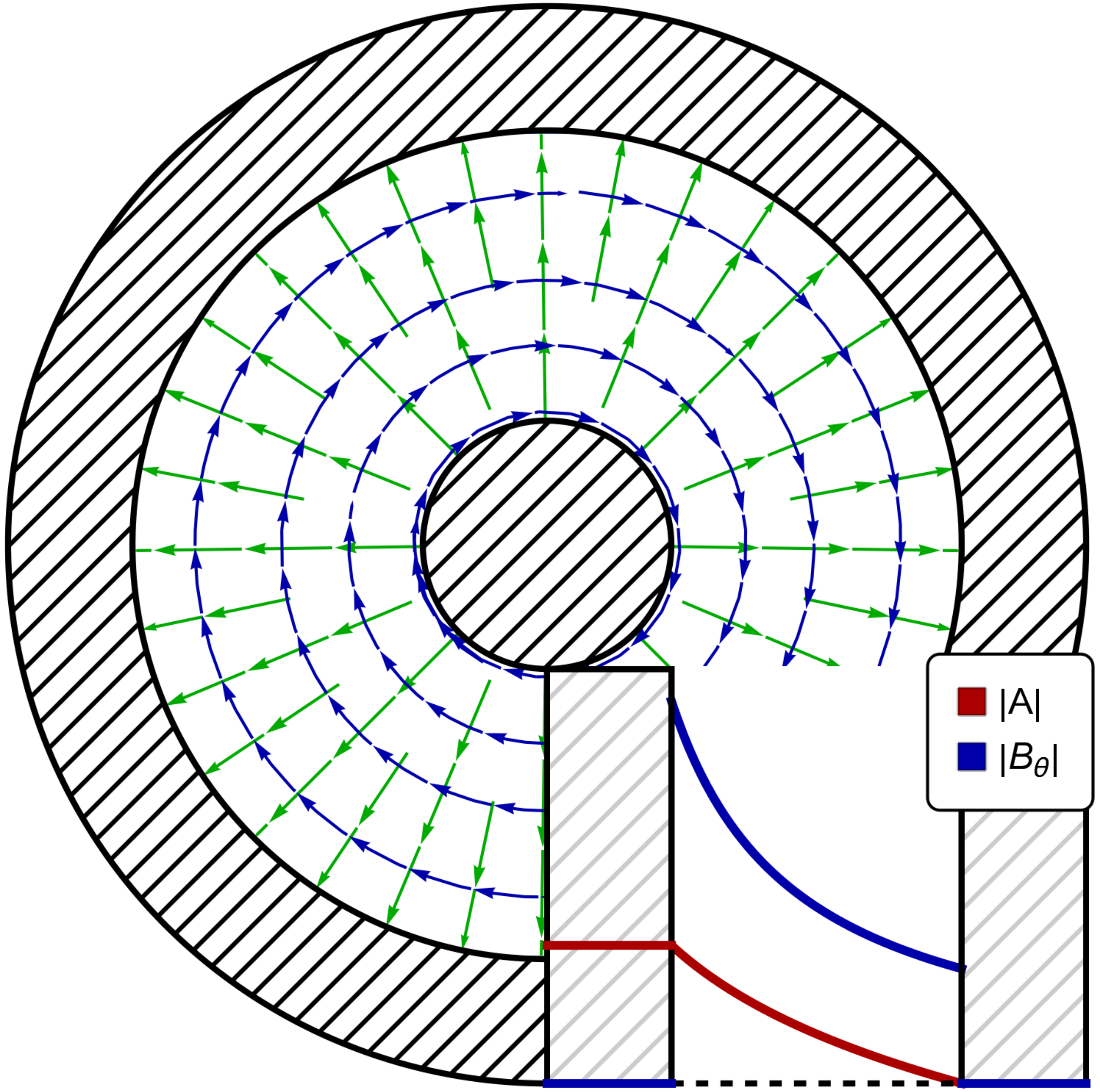
$\Longrightarrow$

Mode solutions:

TM: $B_z = 0$ in U

TE: $E_z = 0$ in U

TEM: $B_z = E_z = 0$ in U



Coaxial Waveguide

Mode solutions:

TM: $B_z = 0 \text{ in } U$

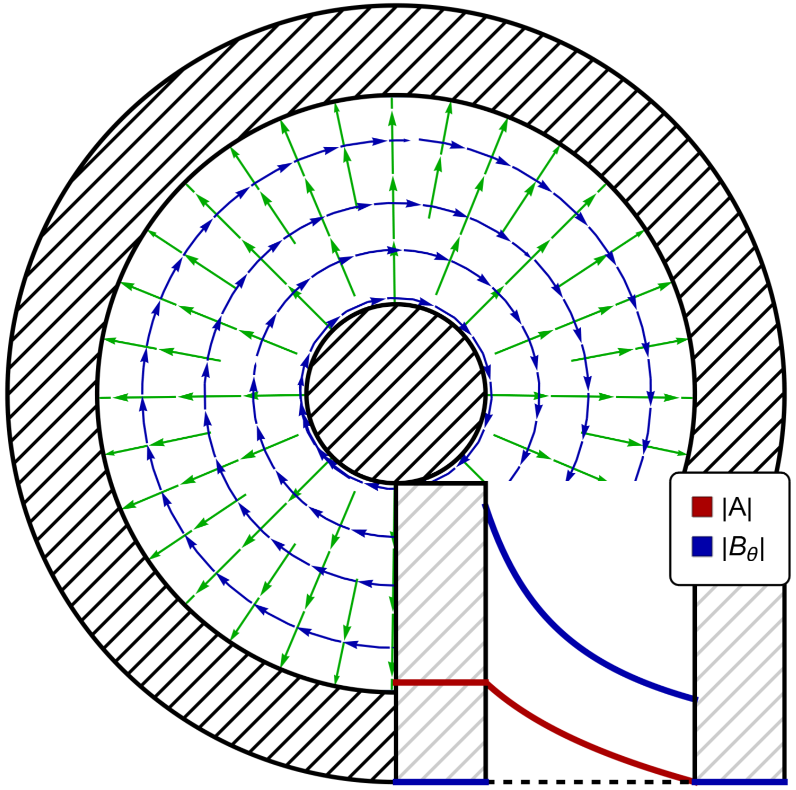
TE: $E_z = 0 \text{ in } U$

TEM: $B_z = E_z = 0 \text{ in } U$

$A_\alpha \neq 0 \text{ on } \partial U$

$\Longrightarrow$

incompatible with Proca



$$\begin{cases} (\Box_\gamma - \mu^2)A_\alpha = 0 & \text{in } U \\ A_\alpha = 0 & \text{on } \partial U \end{cases}$$

Proca in dielectric

Gordon metric: $\gamma^{\alpha\beta} = \eta^{\alpha\beta} + (1 - n^2)u^\alpha u^\beta$

$$\mathcal{L} = -\frac{1}{4}\gamma^{\alpha\rho}\gamma^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + \frac{1}{2}\mu^2\gamma^{\alpha\beta}A_\alpha A_\beta$$

$$\gamma^{\alpha\beta}\partial_\alpha F_{\beta\sigma} + \mu^2 A_\sigma = 0 \quad \Longrightarrow \quad \gamma^{\alpha\beta}\partial_\alpha A_\beta = 0$$

$\mu \ldots$ photon mass
$A \ldots$ vector potential
$u \ldots$ 4-velocity of medium
$n \ldots$ refractive index

Modified Proca in dielectric

$$M^{\alpha\beta} = \eta^{\alpha\beta} + u^\alpha u^\beta - \omega^\alpha \omega^\beta$$

$$\mathcal{L} = -\frac{1}{4}\gamma^{\alpha\rho}\gamma^{\beta\sigma}F_{\alpha\beta}F_{\rho\sigma} + \frac{1}{2}\mu^2 M^{\alpha\beta} A_\alpha A_\beta$$

$$\begin{cases} \Box_\gamma A_\alpha = 0 & \text{longitudinal} \\ \left(\Box_\gamma - \mu^2\right) A_\alpha = 0 & \text{transverse} \end{cases}$$

$\mu\ldots$ photon mass
$A\ldots$ vector potential
$u\ldots$ 4-velocity of medium
$\omega\ldots$ preferred space-like direction

Summary

- Standard formulation of Proca incompatible with TEM modes

  $\longrightarrow$ proposed modified Lagrangian

Outlook

- Extending to optical fiber case

  $\longrightarrow$ estimating photon mass bounds